Strong and weak-universal critical behaviour of a mixed-spin Ising model with a three-spin interaction on the Union Jack lattice

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The mixed spin-1/2 and spin-S Ising model on the Union Jack (centered square) lattice with four different three-spin (triplet) interactions and the uniaxial single-ion anisotropy is exactly solved by establishing a rigorous mapping equivalence with the corresponding zero-field (symmetric) eight-vertex model on a dual square lattice that is exactly soluble due to Baxter [1]. A rigorous proof of the aforementioned exact mapping equivalence is provided in two different independent ways: either by making use of the graph-theoretical formulation of the zero-field eight-vertex model [1] or by employing the spin representation of the zero-field eight-vertex model [2] together with the generalized star-square mapping transformation. The model under investigation generalizes the exactly tractable model originally suggested and examined by Urumov [3] when accounting for the additional uniaxial single-ion anisotropy acting on the spin-S atoms, which reside central sites of each elementary square face of the Union Jack lattice.

The influence of both the interaction anisotropy as well as the uniaxial single-ion anisotropy on phase transitions and critical phenomena is examined in particular. It is shown that the considered model exhibits a strong-universal critical behaviour with constant critical exponents when considering either the isotropic model with four equal triplet interactions or the anisotropic model with at most one different triplet interaction. On the other hand, we have found an exact evidence that the models with the four triplet interactions, which are pairwise equal to each other, exhibit a weak-universal critical behaviour characterized by continuously varying critical exponents. Under these circumstances, the relevant critical exponents are changing along the critical lines in dependence on a relative strength of the triplet interactions as well as the uniaxial single-ion anisotropy. Besides, it is demonstrated that the mixed-spin Ising models with the integer-valued spins S exhibit very different variations of the critical exponents in comparison with the analogous mixed-spin Ising models with the half-odd-integer spins S. The obtained rigorous results will be also confronted with the previously published exact results for the spin-1/2 Ising model with the triplet interaction on a triangular, kagome, decorated triangular and centered square lattice.

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